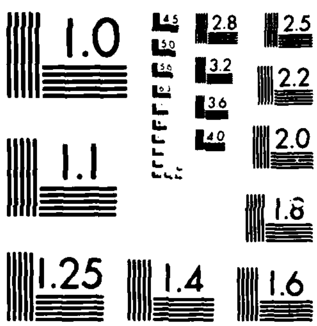
AD-A163 651 THE CLASSIFICATION OF SOLUTIONS OF QUADRATIC RIEMANN 1/1
PROBLEMS (II)(U) WISCONSIN UNIV-MADISON MATHEMATICS
RESEARCH CENTER E ISAACSON ET AL. DEC 85 MRC-TSR-2892
UNCLASSIFIED DAAG29-80-C-0041 F/G 12/1 NL

END
FILMED
DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963 A

(2)

AD-A163 651

MRC Technical Summary Report #2892

THE CLASSIFICATION OF SOLUTIONS OF QUADRATIC RIEMANN PROBLEMS (II)

E. Isaacson and B. Temple

Mathematics Research Center
University of Wisconsin—Madison
610 Walnut Street
Madison, Wisconsin 53705

December 1985

(Received September 23, 1985)

Sponsored by

U.S. Army Research Office
P. O. Box 12211
Research Triangle Park
North Carolina 27709

National Science Foundation
Washington, DC 20550

(A)

UNIVERSITY OF WISCONSIN-MADISON
MATHEMATICS RESEARCH CENTER

THE CLASSIFICATION OF SOLUTIONS OF QUADRATIC RIEMANN PROBLEMS (II)

E. Isaacson[*,6,7,8,9,10] and B. Temple[1,2,3,4,5],

Technical Summary Report #2892
December 1985

This system consists of partial differential equations

ABSTRACT

This is one step in a program aimed at classifying solutions of the Riemann problem for 2×2 system of hyperbolic quadratic conservation laws. Such conservation laws approximate a general 2×2 system of conservation laws in a neighborhood of a point at which strict hyperbolicity fails. We give this document the solution for the symmetric systems in Region III of the four region classification of Schaeffer and Shearer. The solution is based on the qualitative shape of the integral curves described by Schaeffer and Shearer and a numerical calculation of the Hugoniot loci with their shock types.

AMS (MOS) Subject Classifications: 65M10, 76N99, 35L65, 35L67

Key Words: Riemann problem, non-strictly hyperbolic conservation laws, umbilic points

Work Unit Number 1 (Applied Analysis) and 3 (Numerical Analysis and Scientific Computing)

*Department of Mathematics, University of Wyoming, Laramie, Wyoming, 82071.

[1] Supported in part by FINEP/Brazil Grant No. 4.3.82.017.9.
[2] Supported in part by CNPq/Brazil Grant No. 1.01.10.011/84-ACI.
[3] Supported in part by NSF Grant No. 5274298INT-8415209.
[4] Sponsored by the United States Army under Contract No. DAAG29-80-C-0041.
[5] This material is based upon work supported by the National Science Foundation under Grant No. DMS-8210950, Mod. 1.
[6] Supported in part by NSF Grant No. DMS-831229.
[7] Supported in part by DOE Contract No. DE-AC02-76ER03077.
[8] Supported in part by University of Wyoming Division of Basic Research.
[9] Supported in part by AFOSR Grant No. AFOSR-85-0117.
[10] Supported in part by CNPq/Brazil Grant No. 403039/84-MA.

SIGNIFICANCE AND EXPLANATION

A 2×2 system of conservation laws is a system of partial differential equations of the form

$$\underset{\sim}{u}_t + \underset{\sim}{f}(\underset{\sim}{u})_x = 0 \qquad (1)$$

where $\underset{\sim}{u} = (u,v)$, $\underset{\sim}{f} = f(g)$ are in $R \times R$ and where $x \in R$, $t > 0$. Such equations arise in gas dynamics, elasticity, oil reservoir simulation and other areas of engineering when diffusion is neglected. In solutions of (1), information travels at speeds λ_1 and λ_2 given by the eigenvalues of the matrix $\frac{\partial \underset{\sim}{f}}{\partial \underset{\sim}{u}}$. Since this matrix depends on u, v, the speeds λ_1 and λ_2 depend on the solution, and this leads to the formation of discontinuities called shock waves. The present paper deals with the classification of Riemann problem solutions (solutions which evolve from a single discontinuity at time $t = 0$) near an isolated point at which $\frac{\partial \underset{\sim}{f}}{\partial \underset{\sim}{u}}$ is a multiple of the identity, so that $\lambda_1 = \lambda_2$. Such a singularity has no analogue in the linear theory.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the authors of this report.

THE CLASSIFICATION OF SOLUTIONS OF QUADRATIC RIEMANN PROBLEMS (II)

E. Isaacson*,6,7,8,9,10 and B. Temple1,2,3,4,5

§1. Introduction

This is the second in a series of papers [5] in which we give the solution of the Riemann problem for quadratic conservation laws

$$u_t + \frac{1}{2}\{a_1u^2 + 2b_1uv + c_1v^2\}_x = 0 ,$$

$$v_t + \frac{1}{2}\{a_2u^2 + 2b_2uv + c_2v^2\}_x = 0 ,$$

with initial data

$$\underset{\sim}{u}(x,0) = \begin{cases} \underset{\sim}{u}_L \equiv (u_L,v_L) , & x < 0 \\ \underset{\sim}{u}_R \equiv (u_R,v_R) , & x > 0 \end{cases}$$

where $\underset{\sim}{u} \equiv (u,v)$. This is one of the steps in a program outlined in [6].

Solutions of such conservation laws aproximate the solutions of a general 2×2 system of conservation laws in a neighborhood of an isolated point at which strict hyperbolicity fails.

We use the normal form

$$(1) \qquad \begin{aligned} u_t + \frac{1}{2}\{au^2 + 2buv + v^2\}_x &= 0 , \\ v_t + \frac{1}{2}\{bu^2 + 2uv\}_x &= 0 \end{aligned}$$

*Department of Mathematics, University of Wyoming, Laramie, Wyoming, 82071.

1 Supported in part by FINEP/Brazil Grant No. 4.3.82.017.9.
2 Supported in part by CNPq/Brazil Grant No. 1.01.10.011/84-ACI.
3 Supported in part by NSF Grant No. 5274298INT-8415209.
4 Sponsored by the United States Army under Contract No. DAAG29-80-C-0041.
5 This material is based upon work supported by the National Science Foundation under Grant No. DMS-8210950, Mod. 1.
6 Supported in part by NSF Grant No. DMS-831229.
7 Supported in part by DOE Contract No. DE-AC02-76ER03077.
8 Supported in part by University of Wyoming Division of Basic Research.
9 Supported in part by AFOSR Grant No. AFOSR-85-0117.
10 Supported in part by CNPq/Brazil Grant No. 403039/84-MA.

of Schaeffer and Shearer [12].

We let $\lambda_1(\underset{\sim}{u}) < \lambda_2(\underset{\sim}{u})$ denote the eigenvalues of system (1); we note that for system (1), $\lambda_1 = \lambda_2$ only when $\underset{\sim}{u} = \underset{\sim}{0}$, in which case $\lambda = 0$.

In the first paper [5], we presented the solution of the Riemann problem for the parameter values

$$a > 2 , \quad b = 0 .$$

This corresponds to the symmetric systems in Region IV of [12,5]. In the present work we give the solution of the Riemann problem for the parameter range

$$1 < a < 2 , \quad b = 0 ,$$

which corresponds to the symmetric systems in Region III. We refer to [5] for a detailed discussion of the problem and the notation.

We obtain the shapes and shock types of Hugoniot loci $H(\underset{\sim}{u}_L)$ by means of numerical calculation. This, together with the qualitative features of the integral curves given by [12,2] (see Fig. 1) is the basis for our construction of the solutions. We present the solution of the Riemann problem in a series of diagrams for representative values of $\underset{\sim}{u}_L$.

The qualitative features of the solution diagrams change precisely when $\underset{\sim}{u}_L$ crosses $H(\underset{\sim}{0})$ or crosses a ray $\lambda_1 = 0$. In Regions I-III the Hugoniot locus $H(\underset{\sim}{0})$ consists of three straight lines (called axes), while in Region IV this locus is a single line [12]. For the symmetric systems in Regions I-III (i.e., $a < 2$, $b = 0$), one of the axes is the u-axis and the other two are given by

$$v = \pm \sqrt{2 - a}\, u .$$

The rays $\lambda_1 = 0$ are given by

$$v = \begin{cases} -\sqrt{a}\, u & \text{in the lower half plane} \\ +\sqrt{a}\, u & \text{in the upper half plane .} \end{cases}$$

We give the solution of the Riemann problem for representative values of $\underset{\sim}{u}_L$ in the lower half plane $(v < 0)$ because of the symmetries in system (1) (see [5]). Since the solutions change qualitatively only at the rays $H(\underset{\sim}{0})$ and $\lambda_1 = 0$, we choose a representative value of $\underset{\sim}{u}_L$ from each interior and bounding ray of the sectors (see Fig. 2)

$$A_1 = \{\underset{\sim}{u} : \theta_{12} < \theta < 0\} ,$$

$$A_2 = \{\underset{\sim}{u} : \theta_* < \theta < \theta_{12}\} ,$$

$$A_3 = \{\underset{\sim}{u} : \theta_{34} < \theta < \theta_*\} ,$$

$$A_4 = \{\underset{\sim}{u} : -\pi < \theta < \theta_{34}\} ,$$

where

$$\theta_{12} = \arctan\left(-\sqrt{2-a}\right) ,$$

$$\theta_{34} = \arctan\left(+\sqrt{2-a}\right) - \pi ,$$

$$\theta_* = \arctan\left(-\sqrt{a}\right) .$$

The angles correspond to the axes and the ray $\lambda_1 = 0$, respectively.

A new feature in Region III is that the Hugoniot locus $H(\underset{\sim}{u}_L)$ and the 1-wave curve $W_1(\underset{\sim}{u}_L)$ can be disconnected [5]. This makes the solutions more complicated than those found in Region IV.

In Section 2 we describe the elementary waves and Hugoniot loci, and in Section 3 we present the solution of the Riemann problem.

We wish to acknowledge the assistance of Dan Marchesin and Bradley Plohr with the numerical aspects of the problem. We also thank Michael Shearer and David Schaeffer for helpful discussions on many aspects of quadratic Riemann problems. Finally, we would like to thank Dan and Paulo Paes-Leme for their gracious hospitality during our stay in Brazil.

§2. Elementary Waves

The rarefaction waves of system (1) are obtained from the integral curves of the eigenvector fields of the Jacobian $A(\underset{\sim}{u})$. The shock waves of system (1) are obtained from the Hugoniot loci. The general solution of the Riemann problem that we construct is obtained by composing these waves. The solution consists of a 1-composite wave followed by a 2-composite wave where a p-composite wave is a succession of shock and rarefaction waves of the p family [5]. The p-wave curve $W_p(\underset{\sim}{u}_L)$ consists of all final states in p-composite waves with initial state $\underset{\sim}{u}_L$.

The qualitative shapes of the integral curves for the symmetric systems (1) in Region III are depicted in Figure 1 (see [12,2]).

2.1 Hugoniot Loci

The structure of the Hugoniot loci is depicted in Figs. 3A-J for representative values of $\underset{\sim}{u}_L$. These figures indicate the general topological structure of the loci as well as the location of shock types. As $\underset{\sim}{u}_L$ varies, these features change qualitatively only at the boundaries of the Sectors $A_1 - A_4$ (and at $\lambda_2 = 0$). The topological structure of the Hugoniot locus changes at the axes, while at the lines $\lambda = 0$ the topological structure remains fixed but the location of shock types changes. (We do not include $\lambda_2 = 0$ as a boundary since the change in shock types at $\lambda_2 = 0$ does not affect the Riemann problem solutions. Specifically, the wave curves, and hence solutions, do not change qualitatively at $\lambda_2 = 0$.)

Note that the Hugoniot loci of states in quadrant IV of the u,v-plane determine all Hugoniot loci because the reflection of a Hugoniot locus about either the u- or v-axis is again a Hugoniot locus [5]. For clarity, we include the Hugoniot loci for sector A_4 and its boundary rays $\theta = -\pi$, $\theta = \theta_{34}$ because these are relevant to the solution of the Riemann problem.

2.2 Special Points

In Figs. 4B-G, E is a fixed point on the positive u-axis, and $\underset{\sim}{u}_L$ is taken on the integral curve through E. The point A is the limit of the intersection of $(\underset{\sim}{u})$ with the positive u-axis as $\underset{\sim}{u}$ tends to E through states with $v \neq 0$.

In Figs. 3C-F and 4C-F, the points B_L and D_L are the points on $(\underset{\sim}{u}_L)$ at which

$$\sigma(\underset{\sim}{u}_L, B_L) = \lambda_1(\underset{\sim}{u}_L) = \sigma(\underset{\sim}{u}_L, D_L).$$

Necessarily, $\{u_L, D_L, B_L\}$ is a triple shock [5].

In Figs. 3D-I and 4D-I, the points C_L^1 and C_L^2 are the points at which (u_L) is tangent to a 2-integral curve.

3. Solution of the Riemann Problem

In Figs. 4A-J we give the solution of the Riemann problem for the symmetric systems (1) in Region III. In each diagram, $\underset{\sim}{u}_L$ is fixed and an arbitrary point represents $\underset{\sim}{u}_R$. Figs. 4C, E, G and I depict the solutions for $\underset{\sim}{u}_L$ in Sectors A_1, A_2, A_3 and A_4, respectively, in each of which the solution diagrams are qualitatively the same. For completeness, we include solution diagrams for $\underset{\sim}{u}_L$ on the boundaries of these sectors: The ray $\theta = \theta_{12}$ is the axis separating Sectors A_1 and A_2, and the ray $\theta = \theta_{34}$ is the axis separating Sectors A_3 and A_4. The ray $\theta = \theta_*$ which separates Sectors A_2 and A_3 is the key ray on which $\lambda_1 = 0$.

In each solution diagram, the 1-wave curve $W_1(\underset{\sim}{u}_L)$ consists of the union of the 1-shock, 1-rarefaction, and 1-composite curves for $\underset{\sim}{u}_L$. (See the Legend.) For each point $\underset{\sim}{u}_m \in W_1(\underset{\sim}{u}_L)$, the portion of the 2-wave curve $W_2(\underset{\sim}{u}_m)$ appearing in the diagrams is obtained by proceeding from $\underset{\sim}{u}_m$ along 2-shock and 2-rarefaction curves as far as possible in the direction of the arrows.

The solution of the Riemann problem $\langle \underset{\sim}{u}_L, \underset{\sim}{u}_R \rangle$ consists of a 1-wave with left state $\underset{\sim}{u}_L$ and right state $\underset{\sim}{u}_m$ followed by a 2-wave with left state $\underset{\sim}{u}_m$ and right state $\underset{\sim}{u}_R$ where the intermediate state $\underset{\sim}{u}_m$ is determined as follows: start from $\underset{\sim}{u}_R$ and follow the 2-wave curve backwards from $\underset{\sim}{u}_R$ (opposite the direction of the arrows) until you reach a point $\underset{\sim}{u}_m$ in $W_1(\underset{\sim}{u}_L)$. The state $\underset{\sim}{u}_m$ so constructed satisfies $\underset{\sim}{u}_R \in W_2(\underset{\sim}{u}_m)$ and defines the waves in the solution. This procedure is not well-defined precisely when $\underset{\sim}{u}_R$ is in either the compressive portion of $H(\underset{\sim}{u}_L)$ or the triple shock curve. (The triple shock curve is $[AB_L]$ in Figs. 4C-E and is $[-\underset{\sim}{u}_* A]$ in Figs. 4F, G. There is no triple shock curve in the other figures.) In the case of ambiguity, the solutions are unique in the x,t-plane since then all shock speeds that occur are equal. This ensures continuous dependence of the solution on $\underset{\sim}{u}_L$ and $\underset{\sim}{u}_R$.

Figure Captions

Figure 1: Integral Curves

Figure 2: The Sectors $A_1 - A_4$ and Their Boundaries

Figure 3A: The Hugoniot Locus $H(\underset{\sim}{u}_L)$ for $\underset{\sim}{u}_L = \underset{\sim}{0}$

Figure 3B: The Hugoniot Locus $H(\underset{\sim}{u}_L)$ for $\theta_L = 0$

Figure 3C: The Hugoniot Locus $H(\underset{\sim}{u}_L)$ for $\underset{\sim}{u}_L \in A_1$

Figure 3D: The Hugoniot Locus $H(\underset{\sim}{u}_L)$ for $\theta_L = \theta_{12}$

Figure 3E: The Hugoniot Locus $H(\underset{\sim}{u}_L)$ for $\underset{\sim}{u}_L \in A_2$

Figure 3F: The Hugoniot Locus $H(\underset{\sim}{u}_L)$ for $\theta_L = \theta_*$

Figure 3G: The Hugoniot Locus $H(\underset{\sim}{u}_L)$ for $\underset{\sim}{u}_L \in A_3$

Figure 3H: The Hugoniot Locus $H(\underset{\sim}{u}_L)$ for $\theta_L = \theta_{34}$

Figure 3I: The Hugoniot Locus $H(\underset{\sim}{u}_L)$ for $\underset{\sim}{u}_L \in A_4$

Figure 3J: The Hugoniot Locus $H(\underset{\sim}{u}_L)$ for $\theta_L = -\pi$

Figure 4A: Solution Diagram for $\underset{\sim}{u}_L = \underset{\sim}{0}$

Figure 4B: Solution Diagram for $\theta_L = 0$

Figure 4C: Solution Diagram for $\underset{\sim}{u}_L \in A_1$

Figure 4D: Solution Diagram for $\theta_L = \theta_{12}$

Figure 4E: Solution Diagram for $\underset{\sim}{u}_L \in A_2$

Figure 4F: Solution Diagram for $\theta_L = \theta_*$

Figure 4G: Solution Diagram for $\underset{\sim}{u}_L \in A_3$

Figure 4H: Solution Diagram for $\theta_L = \theta_{34}$

Figure 4I: Solution Diagram for $\underset{\sim}{u}_L \in A_4$

Figure 4J: Solution Diagram for $\theta_L = -\pi$

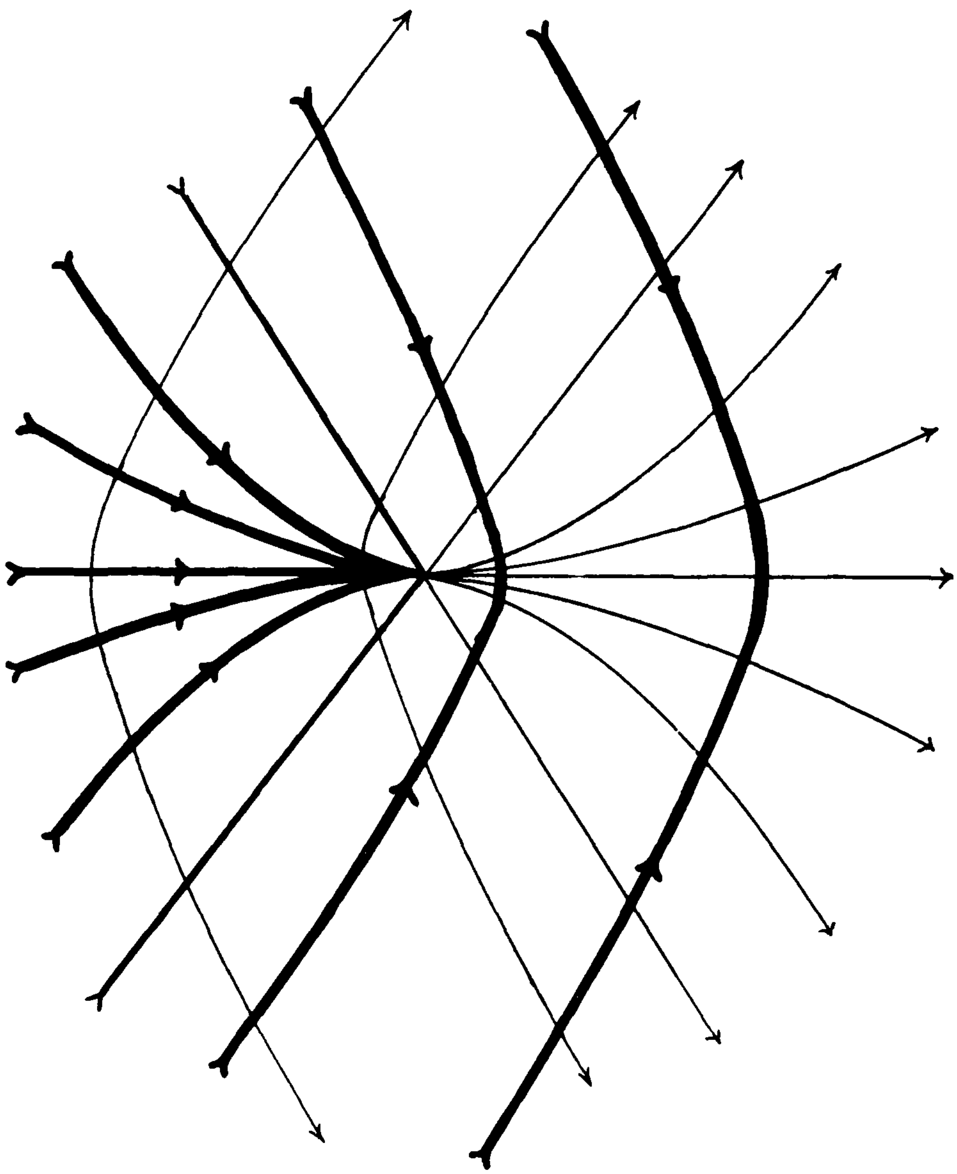


FIGURE 1

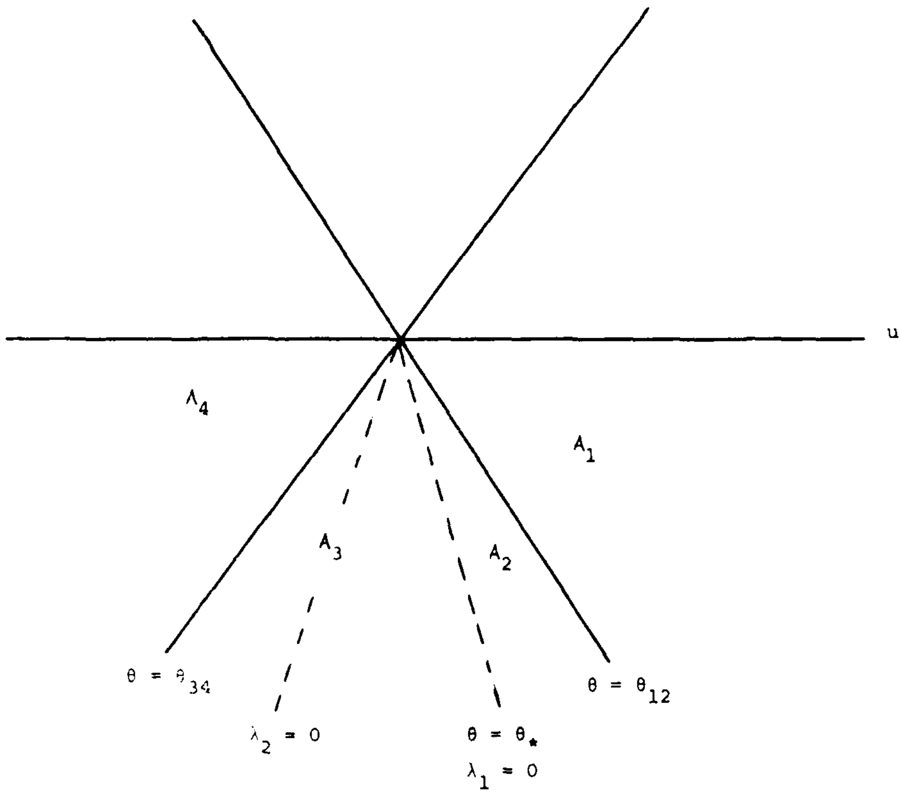

u
A_4
A_1
A_3
A_2
$\theta = \theta_{34}$
$\theta = \theta_{12}$
$\lambda_2 = 0$
$\theta = \theta_*$
$\lambda_1 = 0$

FIGURE 2

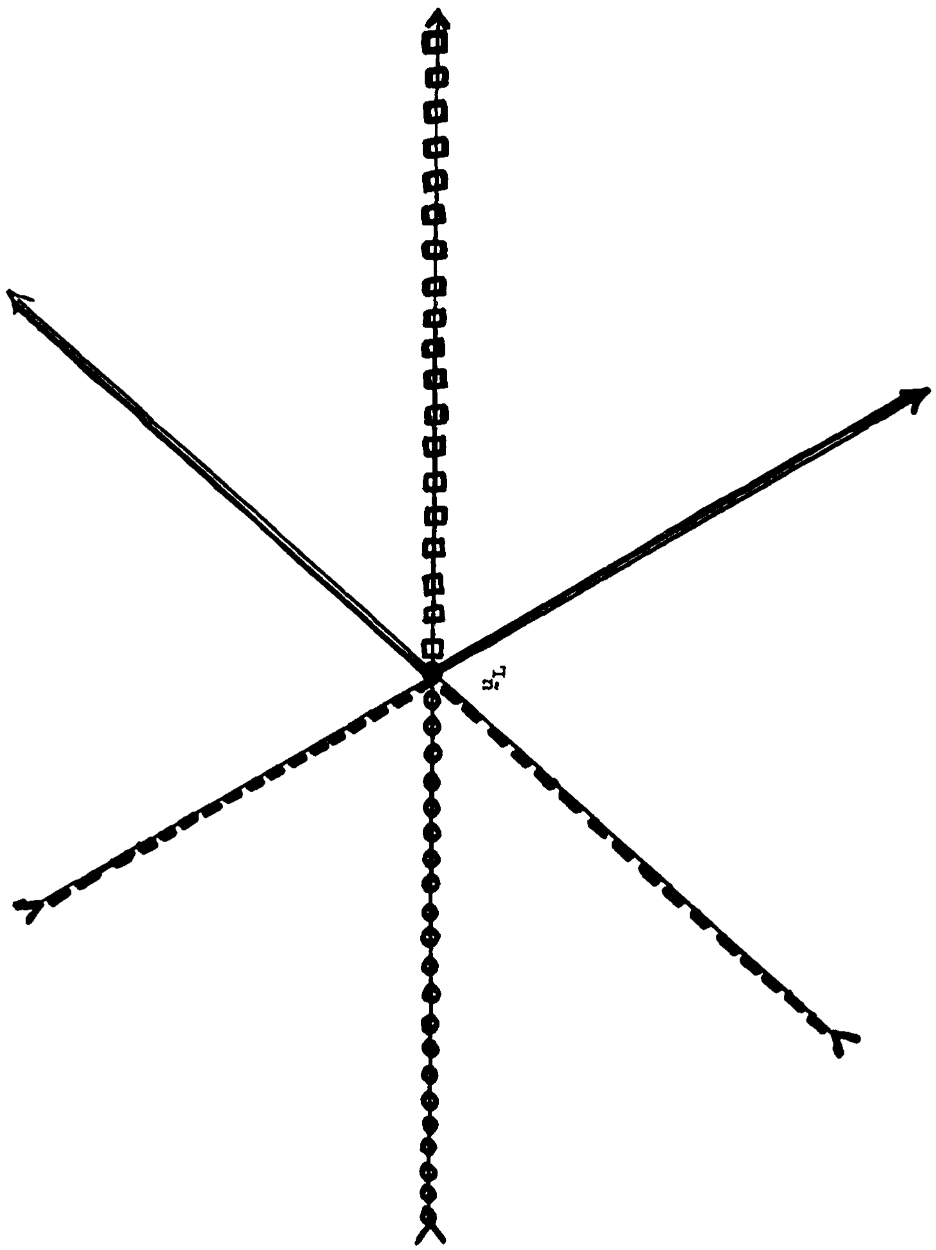


FIGURE 3A

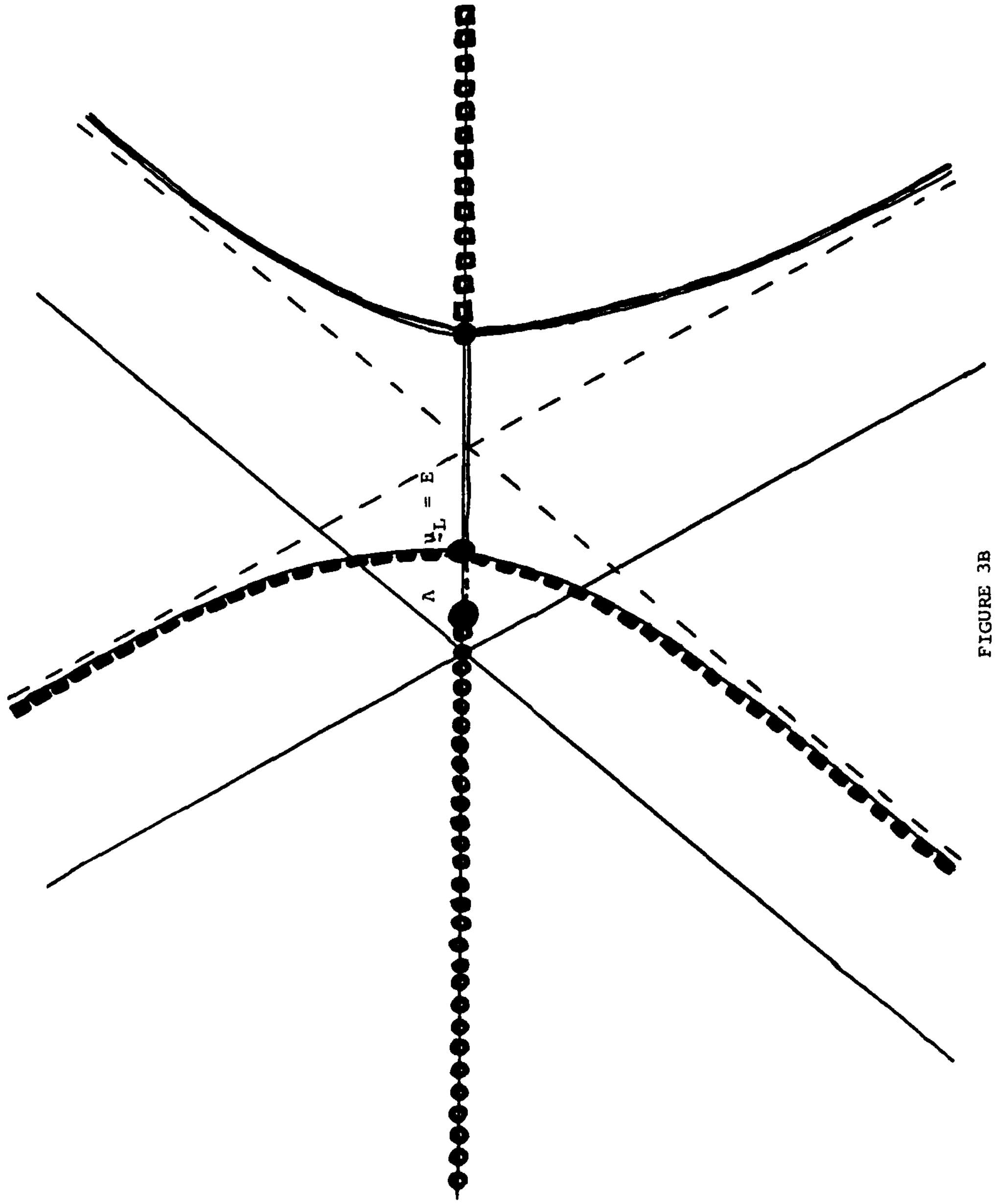


FIGURE 3B

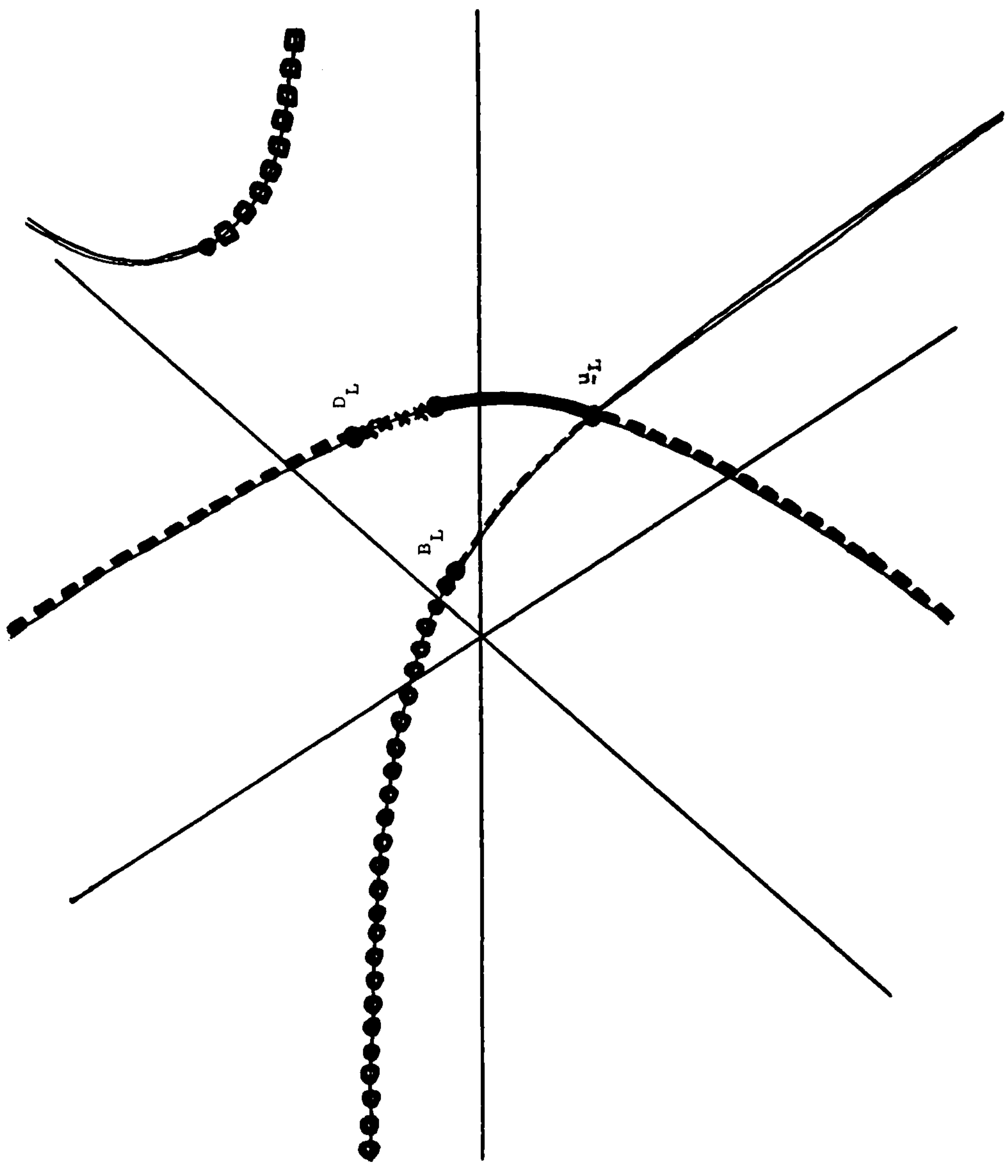


FIGURE 3C

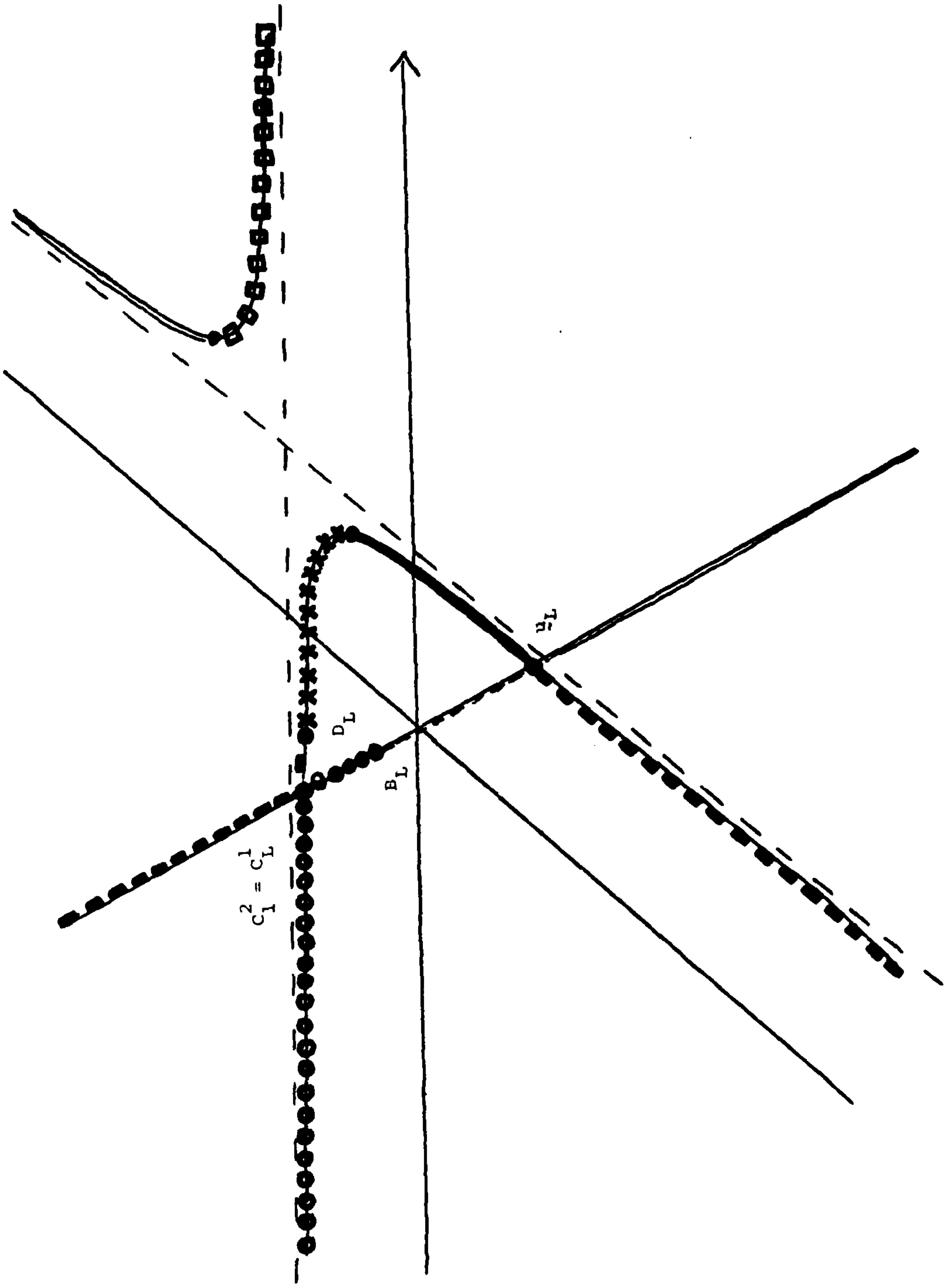


FIGURE 3D

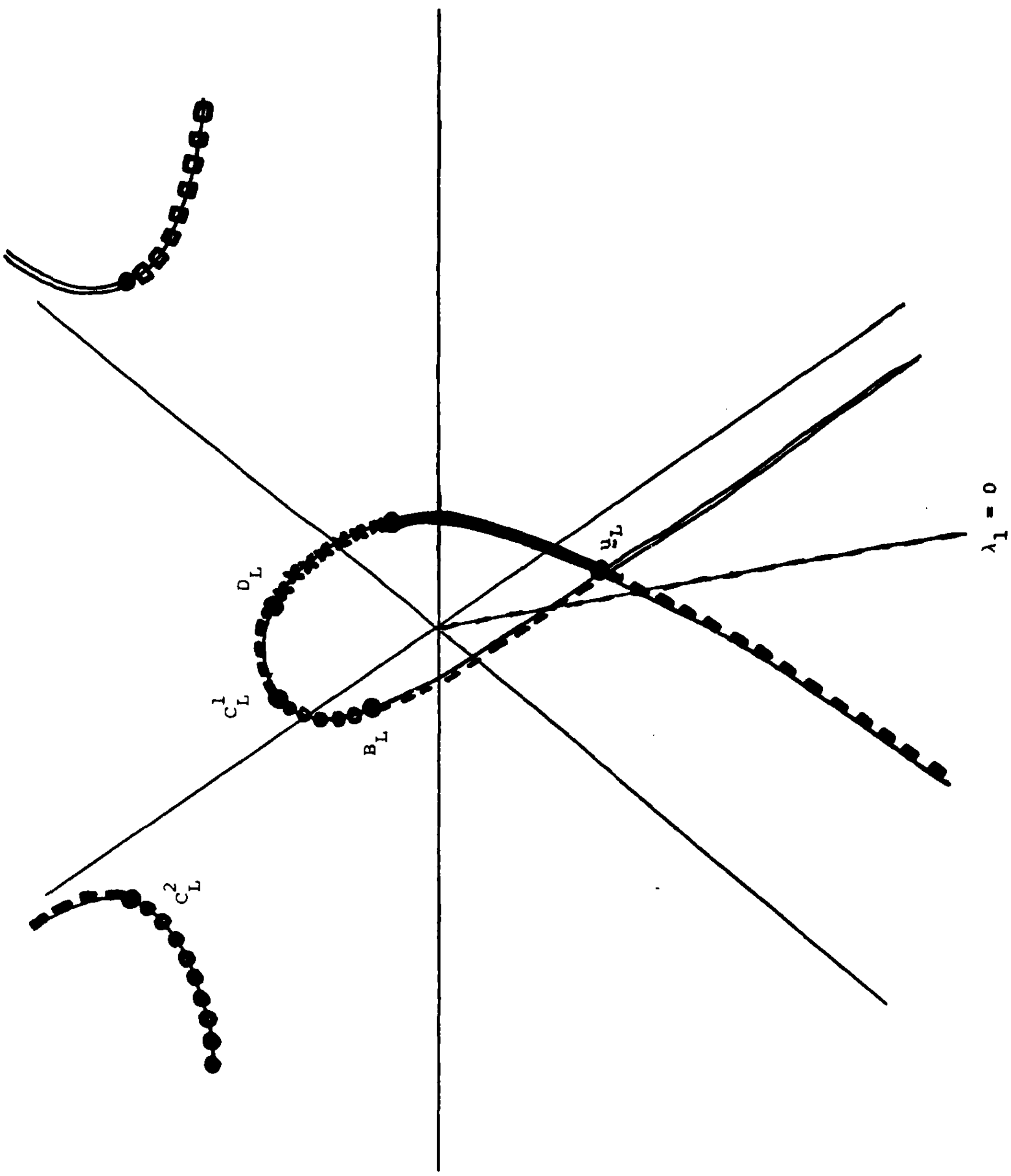


FIGURE 3E

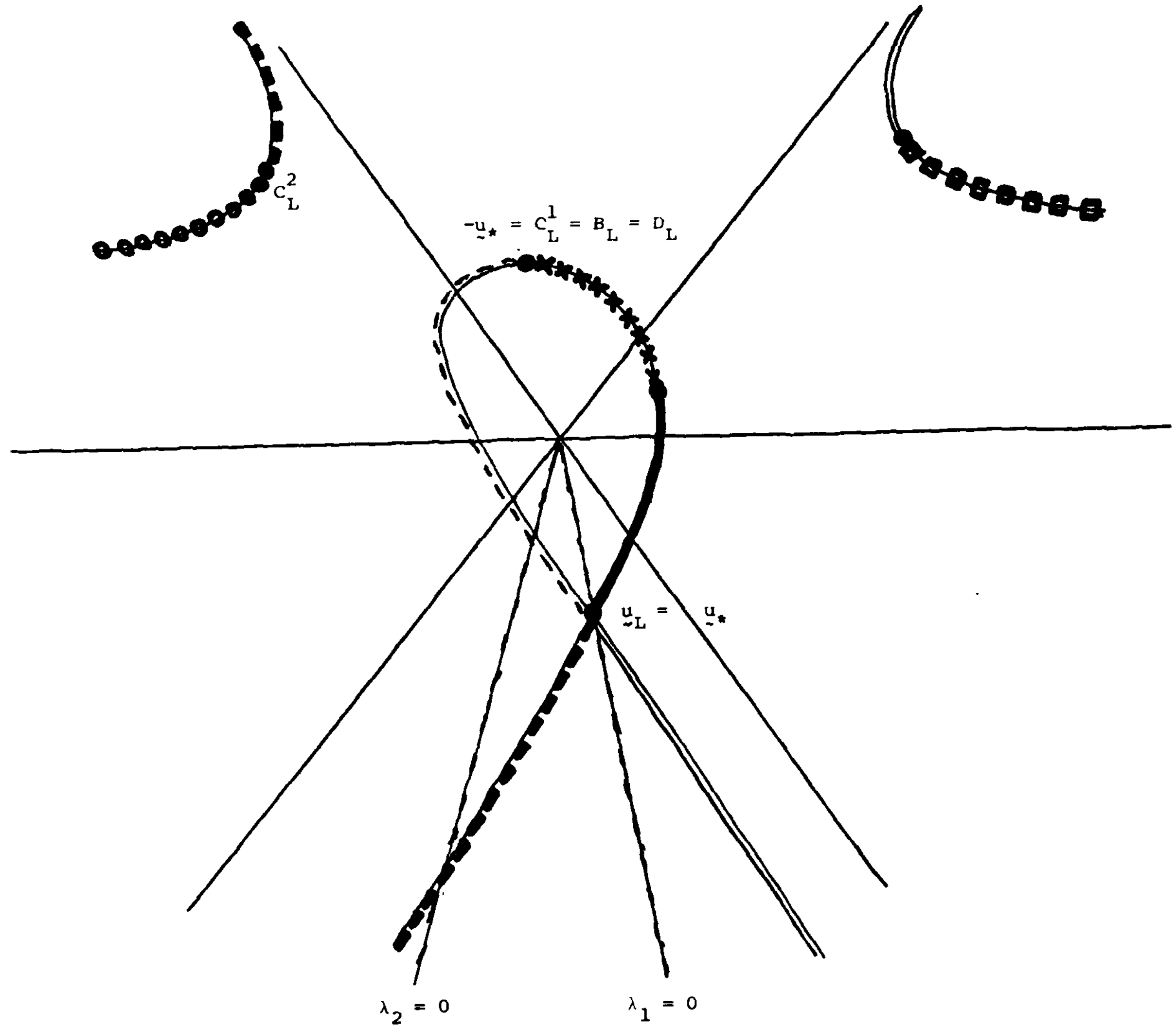

C_L^2
$-\underset{\sim}{u}_* = C_L^1 = B_L = D_L$
$\underset{\sim}{u}_L = \underset{\sim}{u}_*$
$\lambda_2 = 0$
$\lambda_1 = 0$

FIGURE 3F

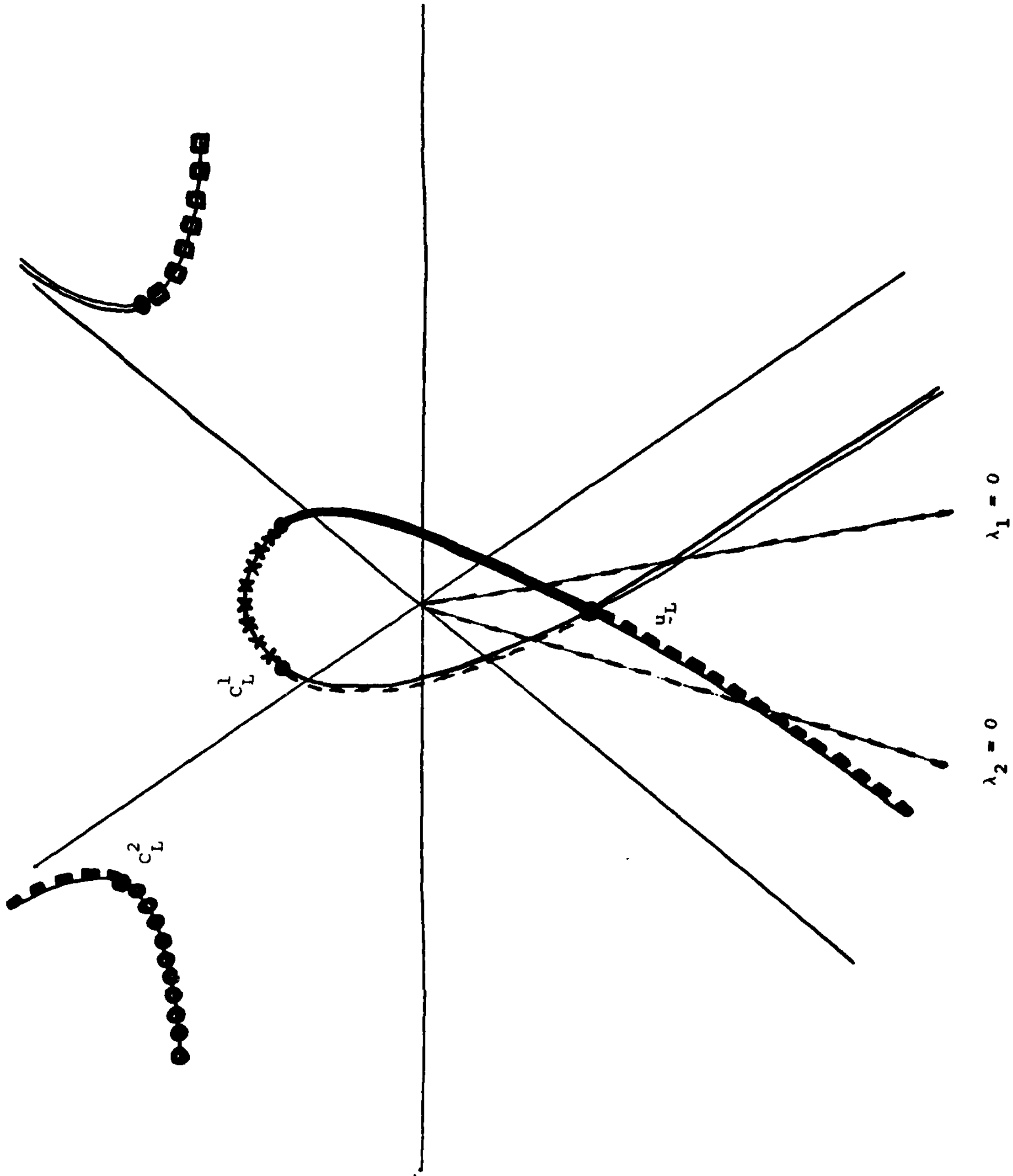


FIGURE 3G

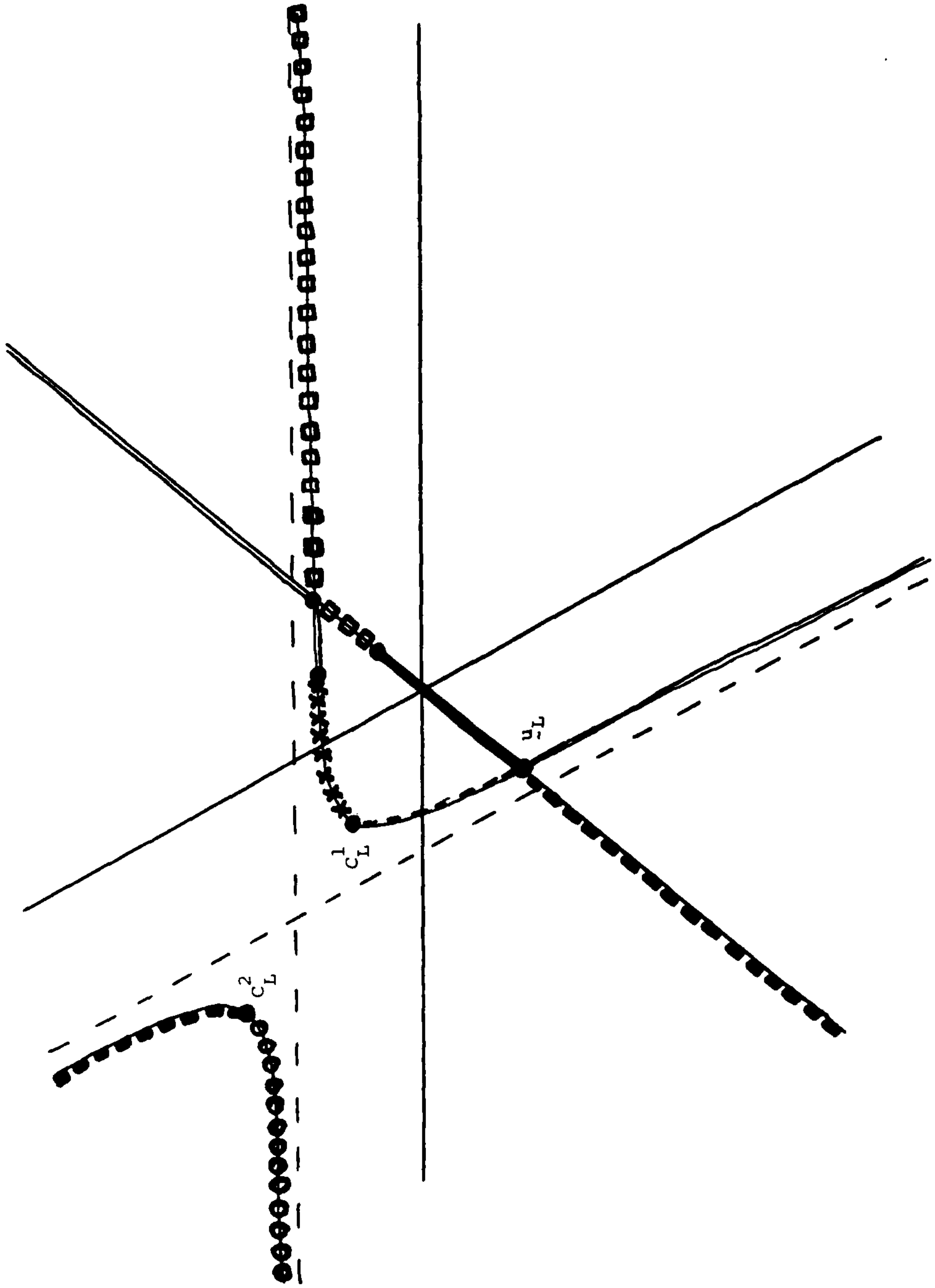


FIGURE 3H

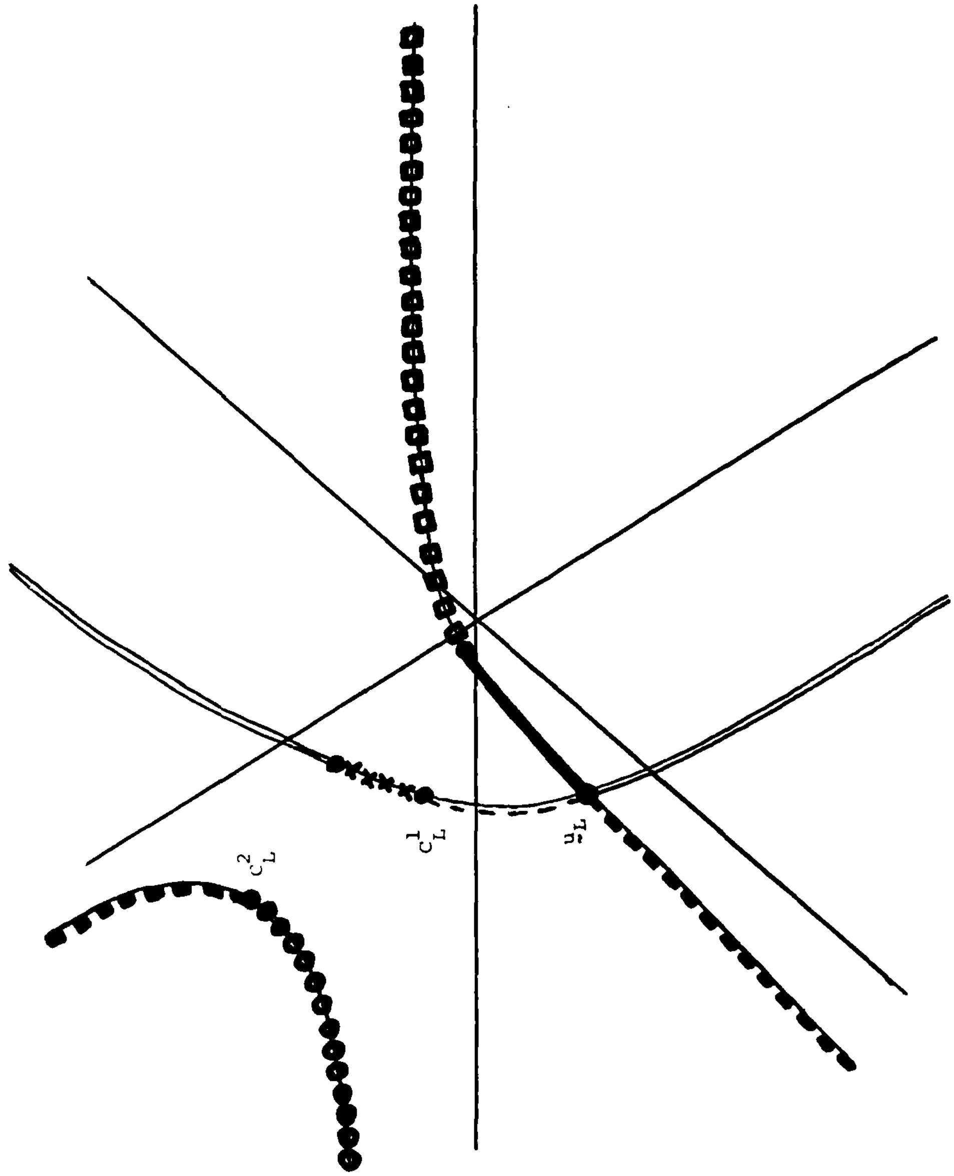


FIGURE 3I

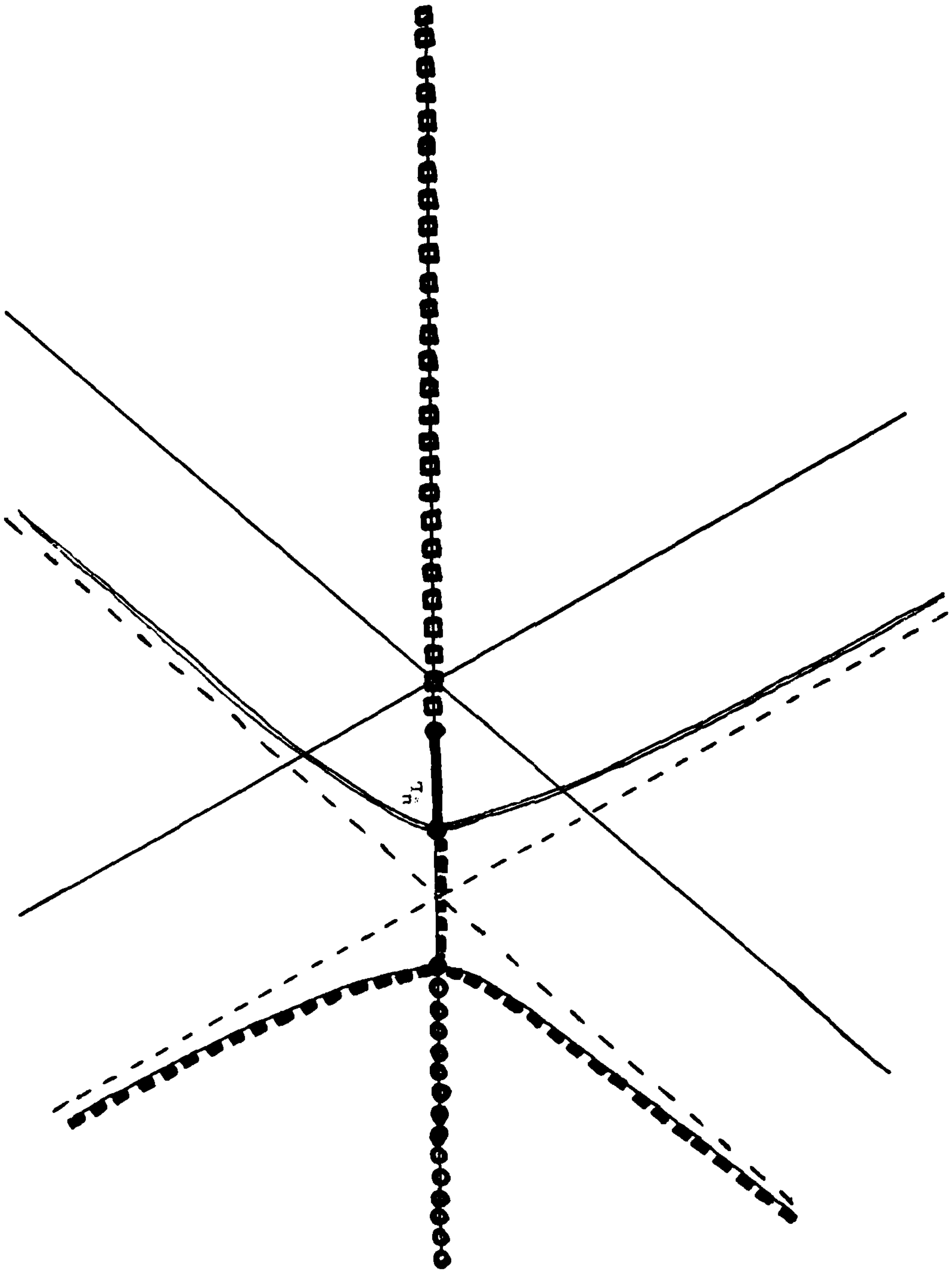


FIGURE 3J

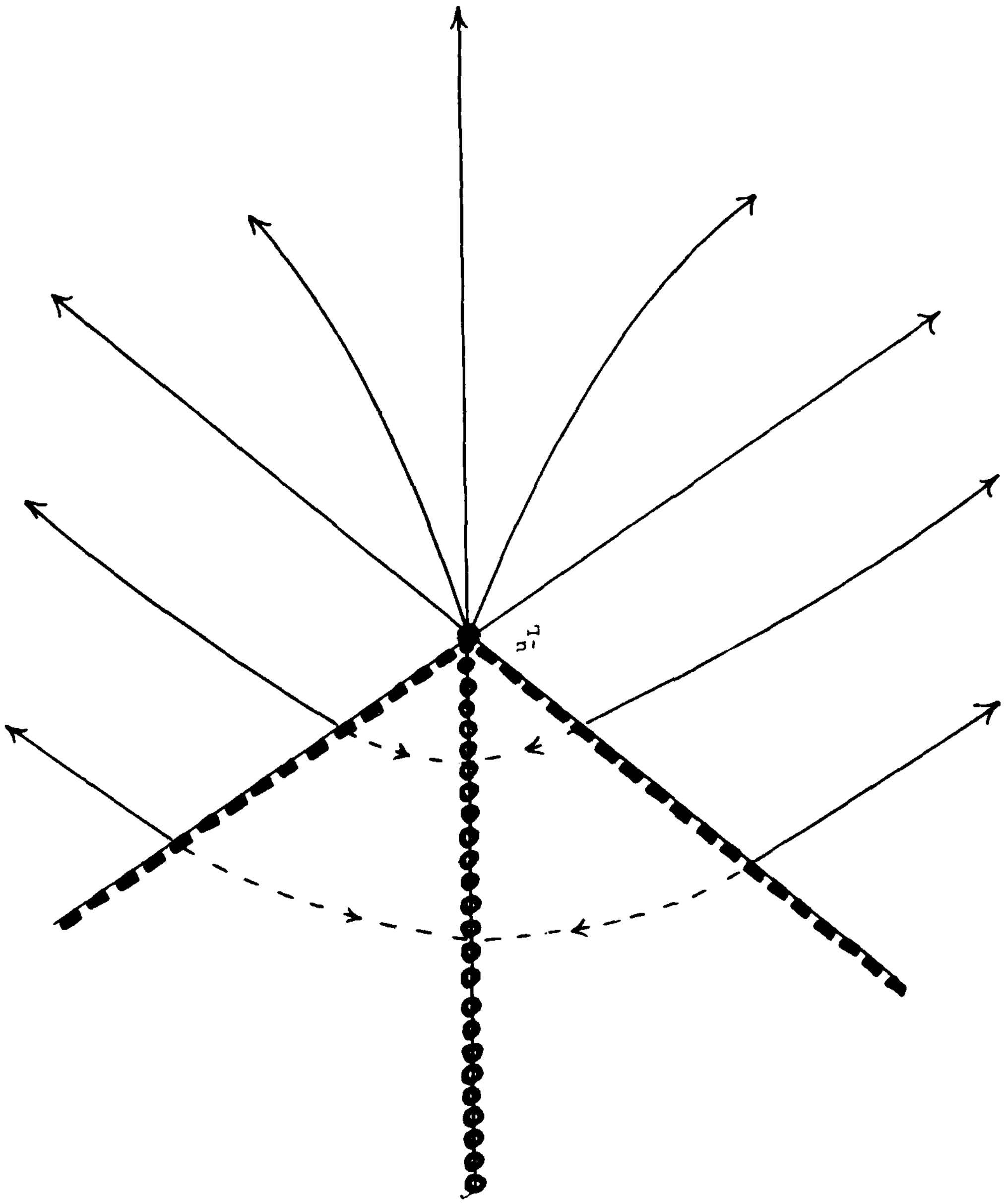


FIGURE 4A

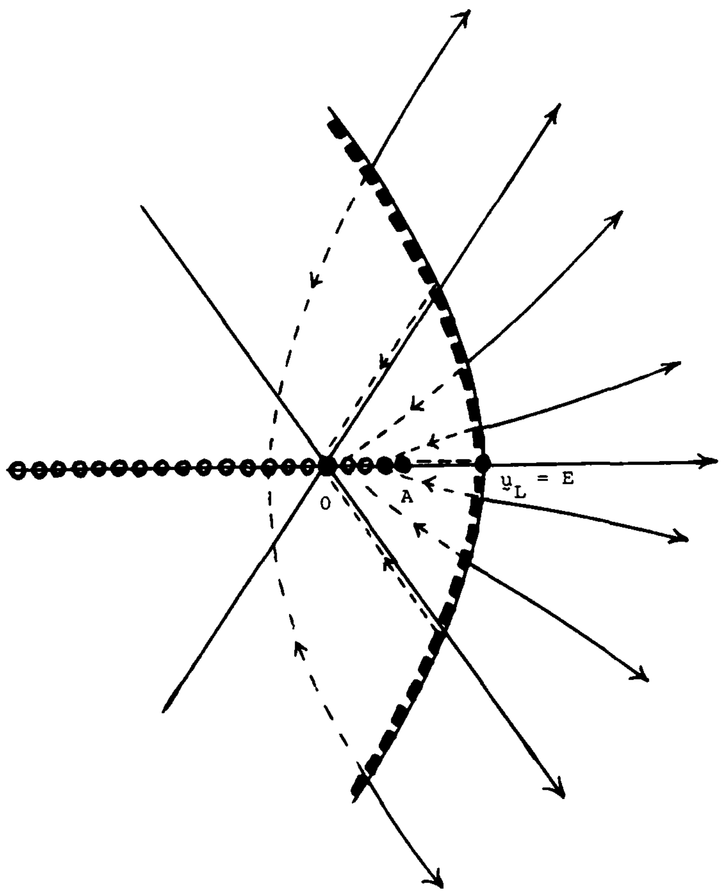


FIGURE 4B

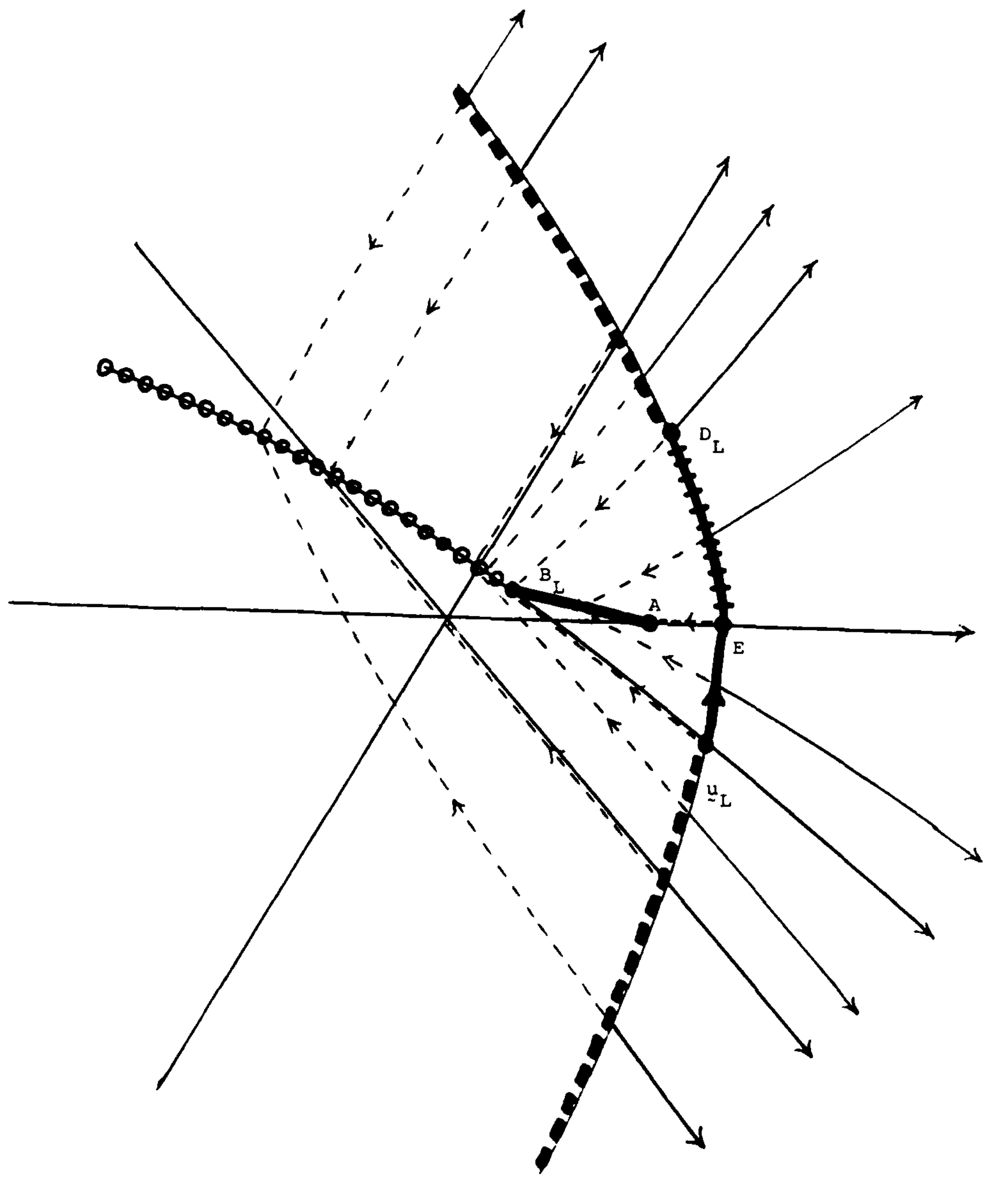


FIGURE 4C

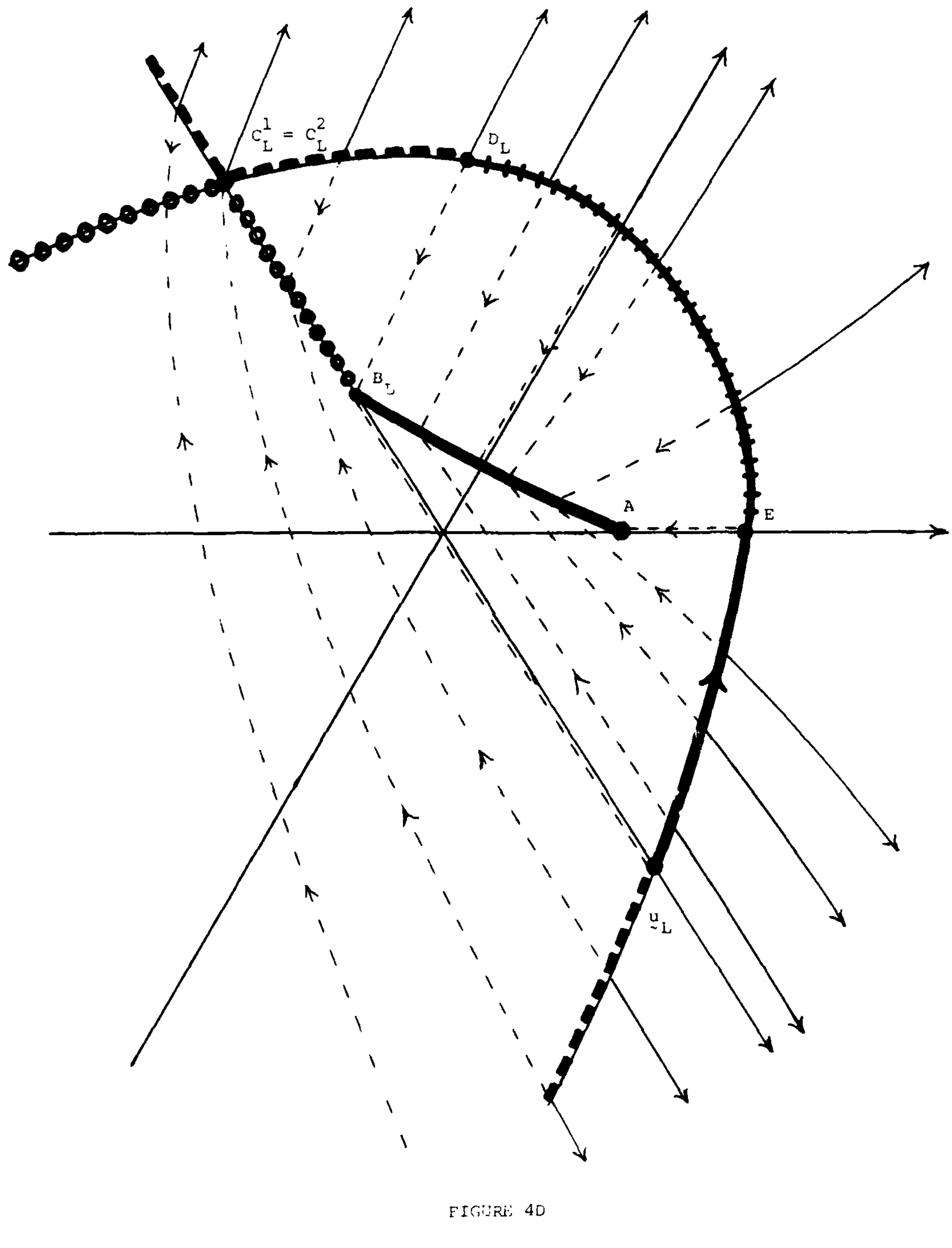


FIGURE 4D

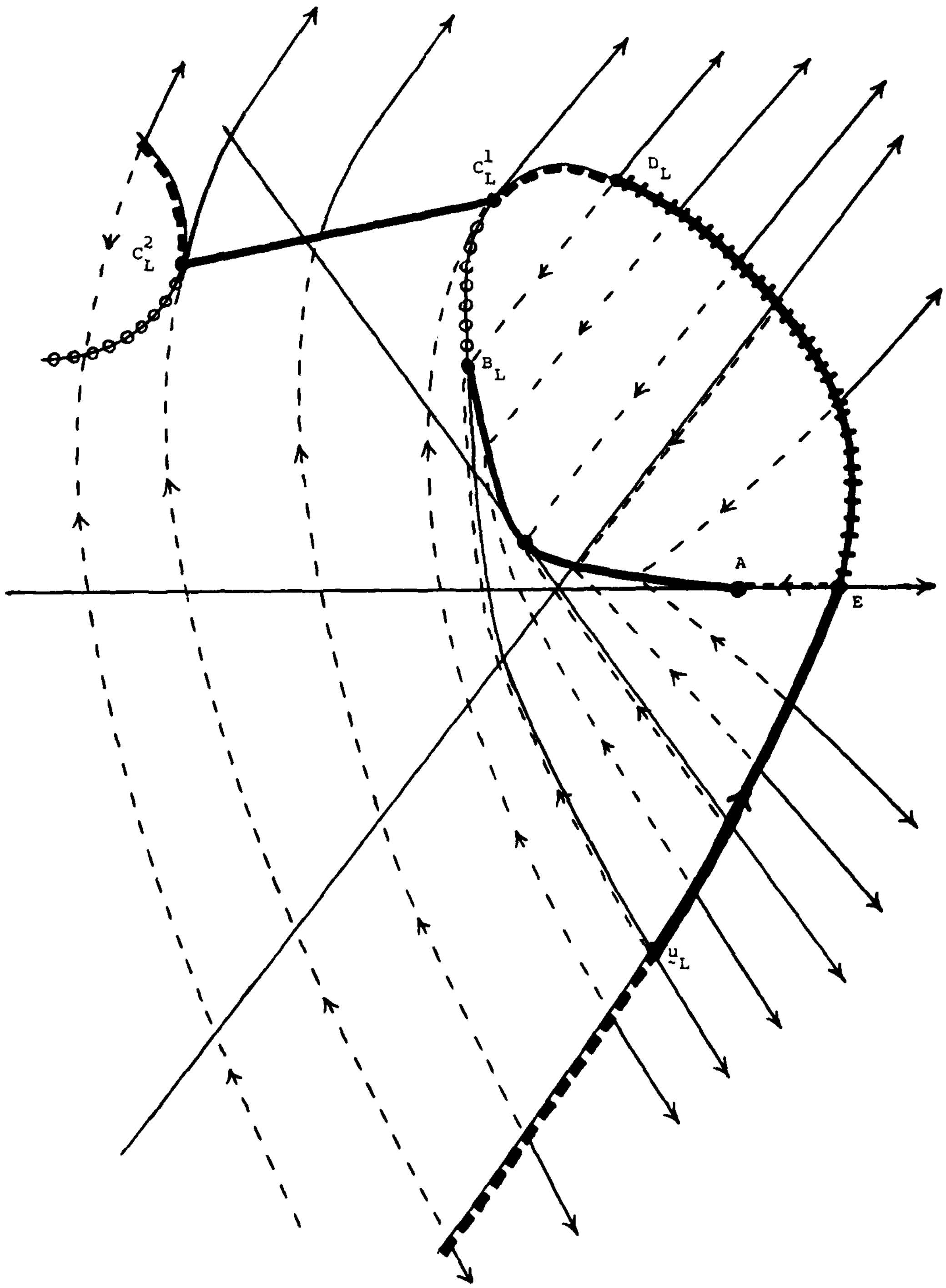


FIGURE 4E

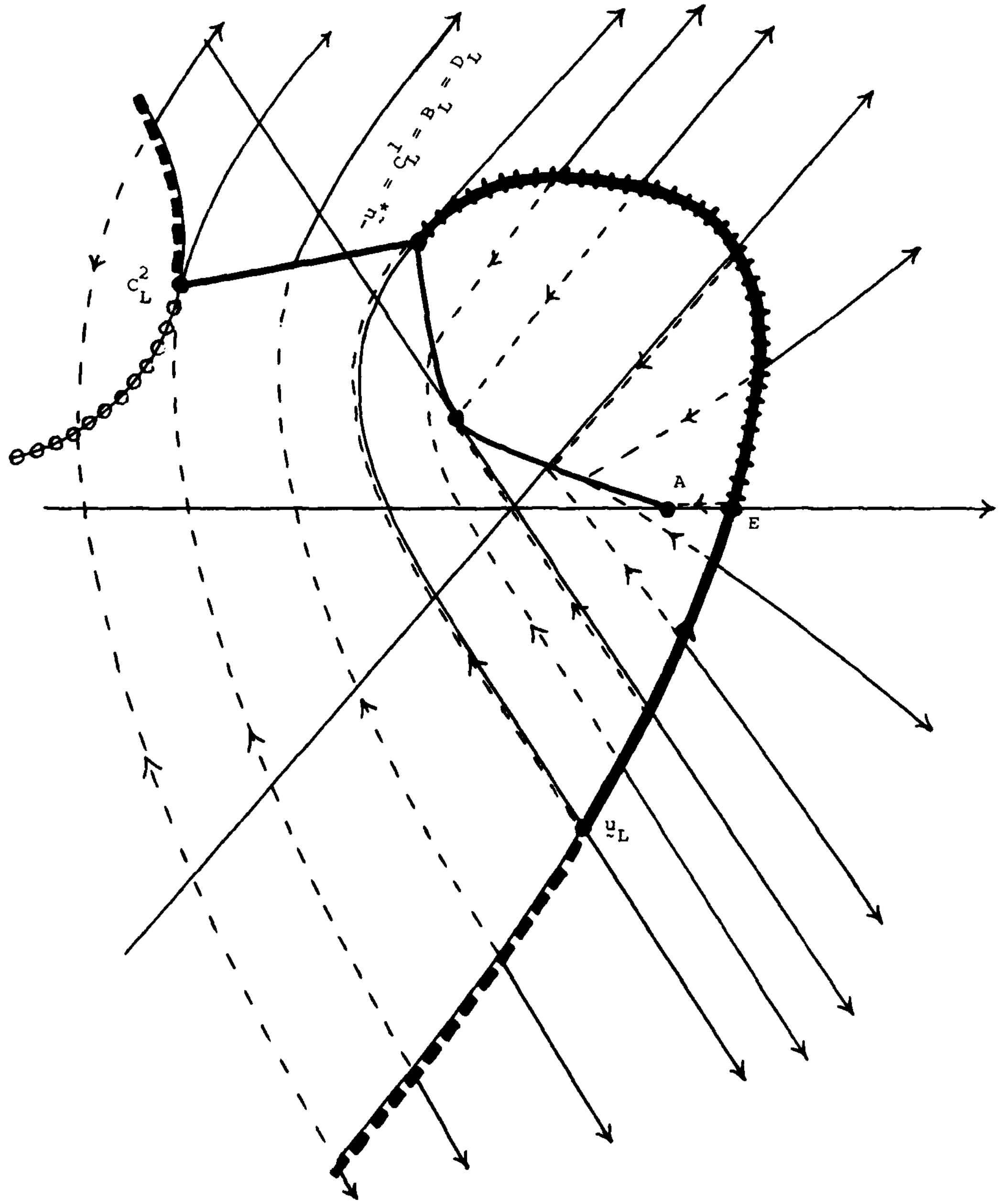


FIGURE 4F

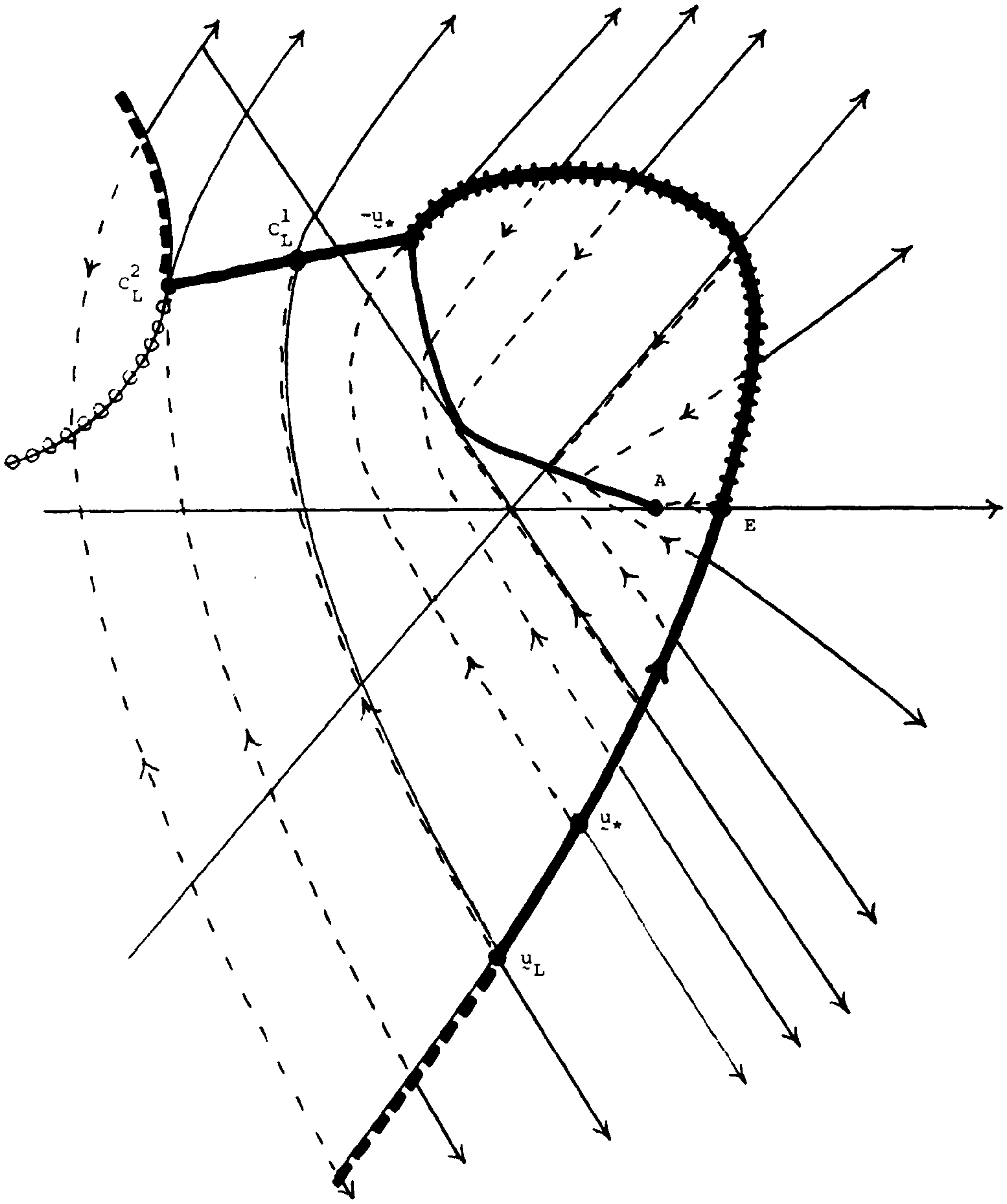


FIGURE 4G

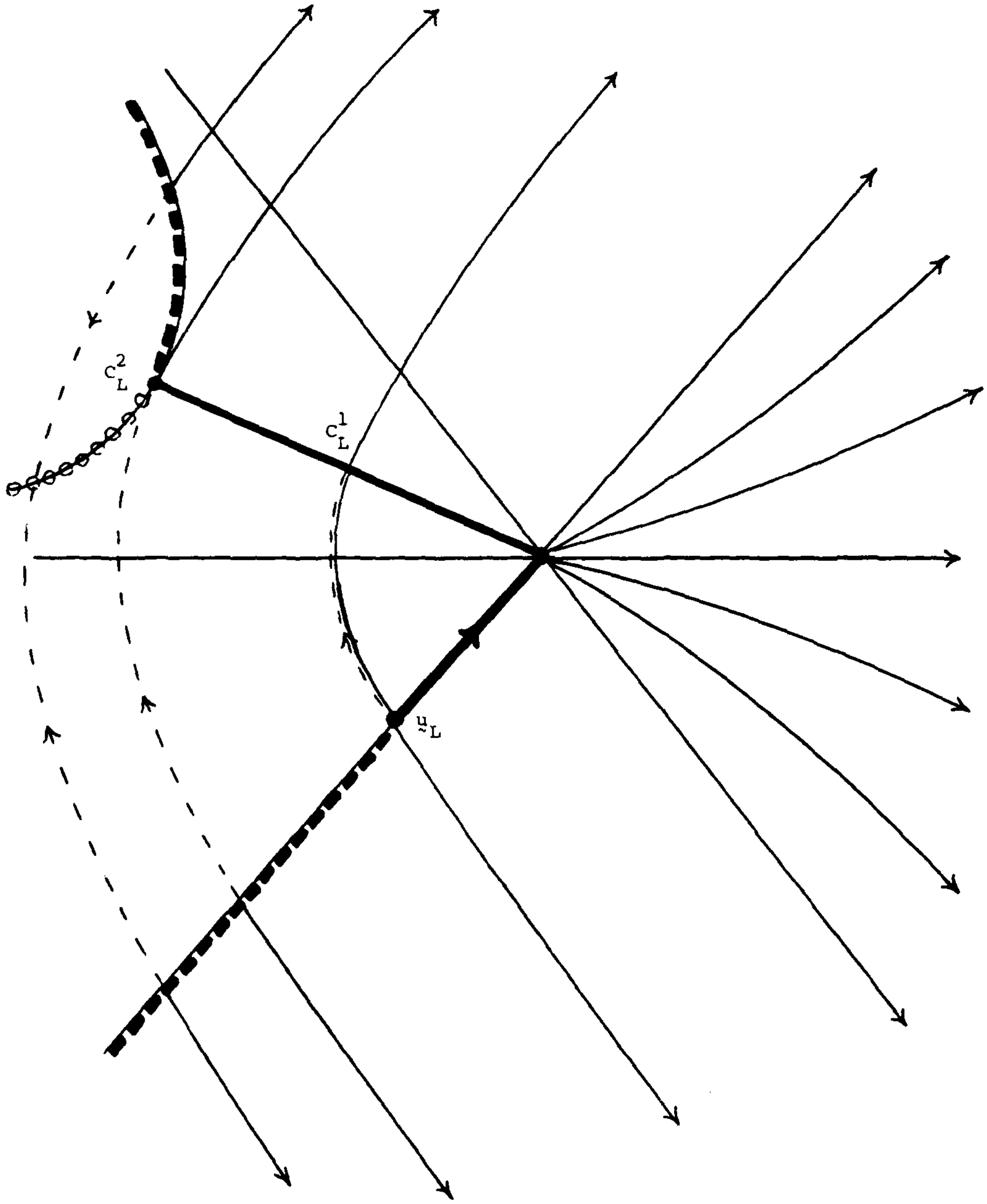


FIGURE 4H

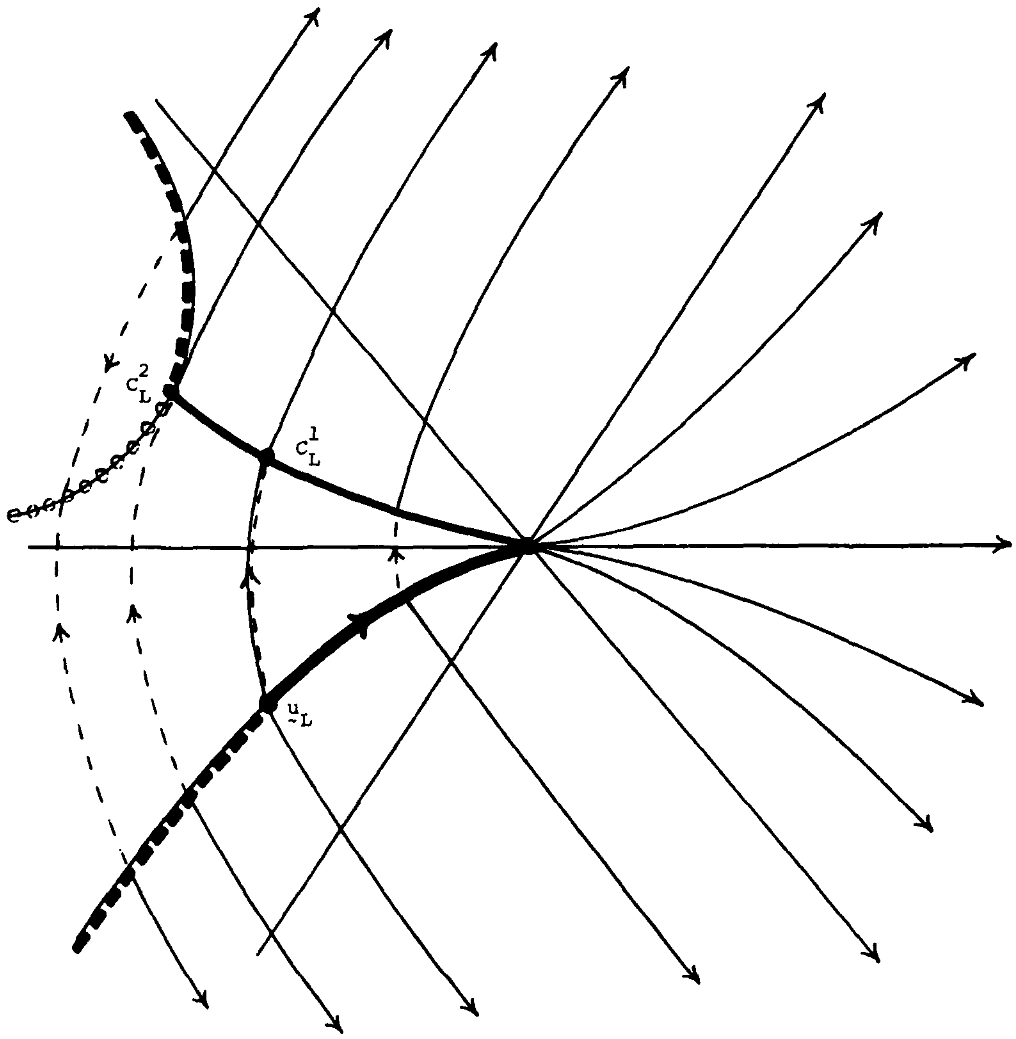


FIGURE 4I

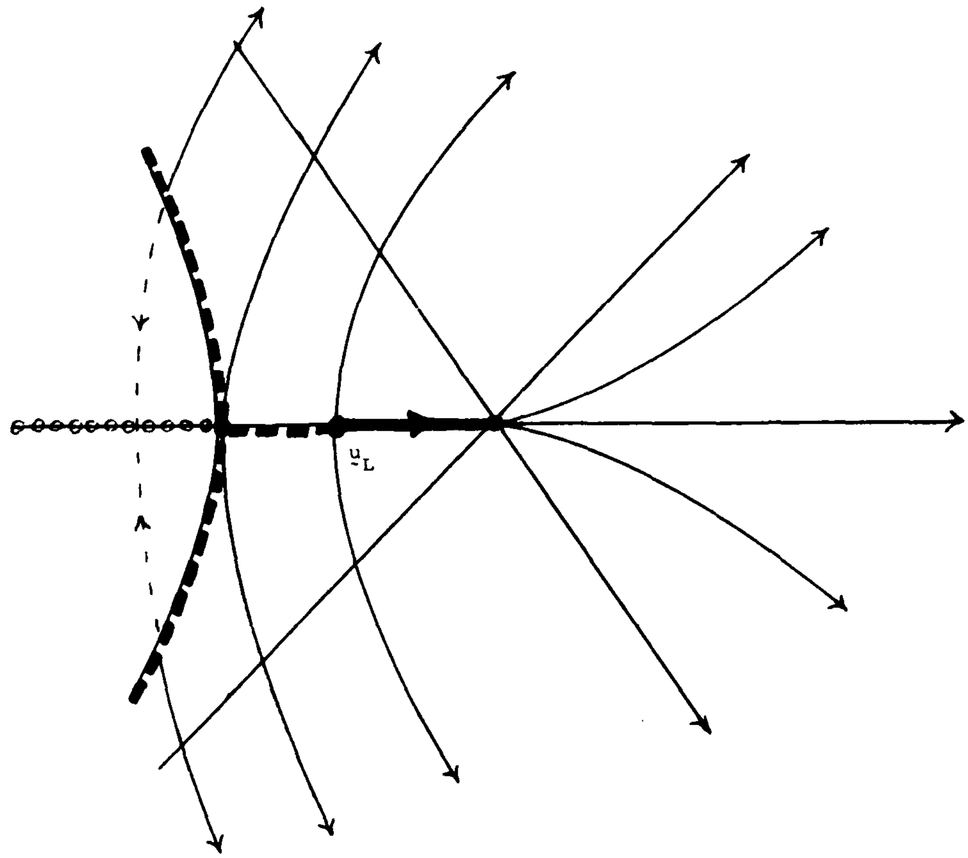


FIGURE 4J

References

[1] Courant, R. and Friedrichs, K. O., Supersonic Flow and Shock Waves, Wiley, New York, 1948.

[2] Darboux, G., Théorie General des Surfaces, Chelsea, New York, 1972.

[3] Isaacson, E., "Global solution of a Riemann problem for a non-strictly hyperbolic system of conservation laws arising in enhanced oil recovery", J. Comp. Phys., to appear.

[4] Isaacson, E. and Temple, B., "Analysis of a singular hyperbolic system of conservation laws", J. Diff. Eqn., to appear.

[5] Isaacson, E., Marchesin, D., Plohr, B., and Temle, B., "The classification of solutions of quadratic Riemann problems (I)", Joint MRC, PUC/RJ Report (1985).

[6] Isaacson, E. and Temple, B., "Examples and classification of non-strictly hyperbolic systems of conservations laws", Abstracts of AMS, Jan., 1985. Presented in the Special Session on "Non-Strictly Hyperbolic Conservation Laws" at the Winter Meeting of AMS, Anaheim, Jan., 1985.

[7] Keyfitz, B. and Kranzer, H., "A system of non-strictly hyerbolic conservation laws arising in elasticity theory", Arch. Rat. Mech. Anal., 72 (1980).

[8] Keyfitz, B. L. and Kranzer, H. C., "The Riemann problem for a class of conservation laws exhibiting a parabolic degeneracy", J. Diff. Eqn. 47 (1983), 35-65.

[9] Lax, P. D., "Hyperbolic systems of conservation laws, II", Comm. Pure Appl. Math., 19 (1957), 537-566.

[10] Lax, P. D., "Shock waves and entropy", in Contributions to Nonlinear Functional Analysis, ed. E. H. Zarantonello, Academic Press, New York, 1971, 643-634.

[11] Liu, T. P., "The Riemann problem for general 2×2 conservation laws", Trans. Amer. Math. Soc., 199 (1974), 89-112.

[12] Schaeffer, D.G. and Shearer, M., "The classification of 2×2 systems of non-strictly hyperbolic conservation laws, with application to oil recovery", with Appendix by D. Marchesin, P. J. Paes-Leme, D. G. Schaeffer, and M. Shearer, preprint, Duke University.

[13] Shearer, M., Schaeffer, D. G., Marchesin, D. and Paes-Leme, P. J. "Solution of the Riemann problem for a prototype 2×2 system of non-strictly hyperbolic conservation laws", preprint, Duke University.

[14] Smoller, J. A., Shock waves and reaction diffusion equations, Springer-Verlag (1980).

[15] Temple, B., "Global solution of the Cauchy problem for a class of 2×2 non-strictly hyperbolic conservation laws", Adv. Appl. Math., 3 (1982), 335-375.

[16] Temple, B., "Systems of conservation laws with coinciding shock and rarefaction curves", Contemporary Mathematics, 17 (1983).

EI:BT:jp/js

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: #2892	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle): The Classification of Solutions of Quadratic Riemann Problems (II)	5. TYPE OF REPORT & PERIOD COVERED: Summary Report - no specific reporting period
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): E. Isaacson and B. Temple	8. CONTRACT OR GRANT NUMBER(s): DMS-831229; 5274298INT-841 DAAG29-80-C-0041 5209 DMS-8210950, Mod. 1
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Mathematics Research Center, University of Wisconsin, 610 Walnut Street, Madison, Wisconsin 53705	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: Work Unit Numbers 1 and 3 - Applied Analysis; Numerical Analysis and Scientific Computing.
11. CONTROLLING OFFICE NAME AND ADDRESS: See Item 18 below	12. REPORT DATE: December 1985
	13. NUMBER OF PAGES: 31
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report): UNCLASSIFIED
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

U. S. Army Research Office
P. O. Box 12211
Research Triangle Park
North Carolina 27709

National Science Foundation
Washington, DC 20550

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Riemann problem, non-strictly hyperbolic conservation laws, umbilic points

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This is one step in a program aimed at classifying solutions of the Riemann problem for 2×2 hyperbolic quadratic conservation laws. Such conservation laws approximate a general 2×2 system of conservation laws in a neighborhood of a point at which strict hyperbolicity fails. We give the solution for the symmetric systems in Region III of the four region classification of Schaeffer and Shearer. The solution is based on the qualitative shape of the integral curves described by Schaeffer and Shearer and a numerical calculation of the Hugoniot loci with their shock types.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

END

FILMED

3-86

DTIC